Motor Design Equations
From
Klinger/Smith Article
And McCreary Comments

9 November, 1997

Beginning with:

$$\gamma = 1.30\left(\frac{T_c}{273}\right)^{-0.032}$$

Where T_c is in degrees K.

$$R = \frac{E}{\gamma\ T_c}$$

Where E is 230 to 289 KJ/Kg for Black Powder and R is stated in J/KgK.

$$P_c = \frac{m_f\ \sqrt{R\ T_c}}{t_b\ b\ A_t}$$

Where m_f is fuel mass in Kg, t_b is burn time in seconds, and:

$$b = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

Burn rate is calculated using:

$$r = a\ P_c^{\,n}$$

With r expressed as m/s. a is r at 1 atmosphere (0.101325 Mpa). Article stated 0.1 Mpa.

$$A_{end} = \frac{\pi\ d^2}{4} = \frac{m_f}{r\ t_b\ \rho}$$

Temperature out:

$$T_{out} = T_c\left(\frac{P_a}{P_c}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

Where P_a is atmospheric pressure and P_c is chamber pressure.

$$Isp = \frac{\sqrt{\frac{2\,\gamma}{\gamma - 1} R \left(T_c - T_o\right)}}{g}$$

Where g is the gravitational constant (9.80665 m/s^2).

$$F_{avg} = \frac{m_f \; g \; Isp}{t_b}$$

$$I_{Tot} = F_{avg} \; t_b = m_f \; g \; Isp$$

McCreary's letter states:

$$N = \frac{0.00001 \, \pi \, d^2 \; a \; \rho \sqrt{R \, T_c}}{4 \, b \, A_t}$$

And for chamber pressure gives:

$$P_c = 10^{\left(\frac{\log N}{1-n}\right)}$$

Article used metric system, my problem is getting chamber pressure to come out right. A simplification of McCreary's N is:

$$N = \frac{0.00001 \, Kn \, a \; \rho \sqrt{R \, T_c}}{b}$$

Another equation for chamber pressure is:

$$P_c = \left(\frac{a \; Kn \; \rho \; c^*}{g} \right)^{\left(\frac{1}{1-n}\right)}$$

This is my trouble area. Units for a must be such that units for g, C^*, and ρ combine with it to give desired units for chamber pressure. If pressure is in pascals, *SPAD* gives chamber pressure as:

$$P_c = \left(a \; Kn \; \rho \, c^*\right)^{\left(\frac{1}{1-n}\right)}$$

SPAD also gives a_c (acoustic velocity in chamber) as:

$$a_c = \sqrt{\gamma \; R \; T_c}$$

and for Γ :

$$\Gamma' = \gamma \left(\frac{2}{\gamma + 1} \right)^{\left(\frac{\gamma + 1}{2(\gamma - 1)} \right)}$$

SPAD then defines c^* as:

$$c^* = \frac{a_c}{\Gamma'}$$

Need to keep working with this until I understand it better.

SPAD = Solid Propulsion Analysis and Design, good book, but again in metric.